The Council has released specimen papers for March 2023 examination. A new examination pattern has been proposed wherein 40 marks are allotted to Section A and 40 marks to Section B. Please note the suggested changes in the number of questions and their marks in the solved specimen paper given below as well as in the Model Question Papers given ahead.

ICSE 2023 EXAMINATION
SPECIMEN QUESTION PAPER
MATHEMATICS

Maximum Marks: 80

Time allowed: Two and half hours

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during first 15 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ]

Mathematical tables are provided.

SECTION A

*(Answer **all** questions from this Section)*

***Question* 1 :**

Choose the correct answers to the questions from the given options : **[15]**

(i) The SGST paid by a customer to the shopkeeper for an article which is priced at ₹ 500 is ₹ 15. The rate of GST charged is :

(a) 1·5% (b) 3% (c) 5% (d) 6%

Ans. (d) 6%

Explanation :

$\because$ Rate of SGST = $\frac{15}{500} \times 100\% = 3\%$

$\therefore$ **Rate of GST** = $2 \times 3\%$ = **6%**

(ii) When the roots of a quadratic equation are real and equal then the discriminant of the quadratic equation is :

(a) Infinite (b) Positive (c) Zero (d) Negative

Ans. (c) Zero

Explanation :

When discriminant $b^2 - 4ac$, for quadratic equation $ax^2 + bx + c = 0$, is zero *i.e.* $b^2 - 4ac = 0$. The equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ will give real and equal values of x.

(iii) If $(x - 1)$ is a factor of $2x^2 - ax - 1$, then the value of 'a' is :

(a) –1 (b) 1 (c) 3 (d) –3

Ans. (b) 1

Explanation :

$x - 1 = 0 \Rightarrow x = 1$

Since $(x - 1)$ is a factor of $2x^2 - ax - 1$

$\Rightarrow$ The value of $2x^2 - ax - 1$ is zero for $x = 1$

$\Rightarrow 2 \times (1)^2 - a \times 1 - 1 = 0 \Rightarrow \boldsymbol{a = 1}$

(iv) Given $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times X = \begin{bmatrix} p \\ q \end{bmatrix}$. The order of matrix X is :

(a) 2 × 2 (b) 1 × 2 (c) 2 × 1 (d) 1 × 1

Ans. (c) 2 × 1

Explanation :

Let the order of matrix X be $m \times n$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} \times X_{m \times n} = \begin{bmatrix} p \\ q \end{bmatrix}_{2 \times 1}$$

$\Rightarrow$ No. of columns of 1st matrix = No. of rows in 2nd matrix

$\Rightarrow 2 = m$

Also, no. of columns in 2nd matrix = No. of columns in 3rd matrix

$\Rightarrow n = 1$

$\therefore$ **Order of matrix X** $= m \times n = \mathbf{2 \times 1}$

(v) 57, 54, 51, 48, ... are in Arithmetic Progression. The value of the 8^{th} term is :

(a) 36 (b) 78 (c) –36 (d) –78

Ans. (a) 36

Explanation :

For the given Arithmetic Progression,

first term, $a = 57$ and common difference $d = -3$

$\therefore$ **8^{th} term** $= a + 7d$

$= 57 + 7 \times (-3) = 57 - 21 = \mathbf{36}$

(vi) The point $A(p, q)$ is invariant about $x = p$ under reflection. The coordinates of it's image A′ is :

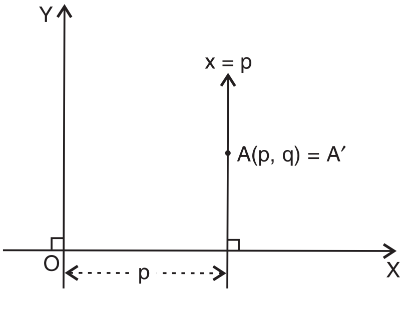
(a) $A'(p, -q)$ (b) $A'(-p, q)$ (c) $A'(p, q)$ (d) $A'(-p, -q)$

Ans. (c) $A'(p, q)$

Explanation :

Under reflection in line $x = p$, the image point A′ coincides with object point $A(p, q)$

$\therefore$ $\mathbf{A' = (p, q)}$



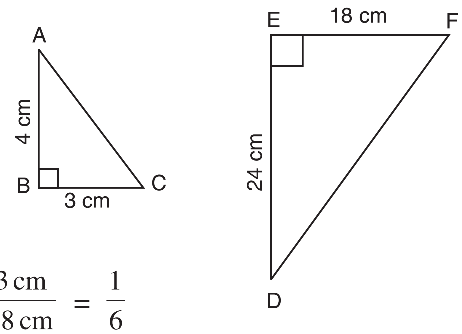
(vii) In the given diagram the ΔABC is similar to ΔDEF by the axiom :

(a) SSS

(b) SAS

(c) AAA

(d) RHS



Ans. (b) SAS

Explanation :

Since $\frac{AB}{DE} = \frac{4\text{ cm}}{24\text{ cm}} = \frac{1}{6}$ and $\frac{BC}{EF} = \frac{3\text{ cm}}{18\text{ cm}} = \frac{1}{6}$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$ *i.e.* two sides of ΔABC are in proportion to two sides of ΔDEF.

Also, their contained angles are equal as $\angle B = \angle E = 90°$

$\Rightarrow$ **ΔABC is similar to ΔDEF by axiom SAS**

(viii) The volume of a right circular cone with same base radius and height as that of a right circular cylinder, is 120 cm^3. The volume of the cylinder is :

(a) 240 cm^3 (b) 60 cm^3 (c) 360 cm^3 (d) 480 cm^3

Ans. (c) 360 cm^3

Explanation :

For the same radius and height, volume of cylinder is $\pi r^2 h$ and volume of cone is $\frac{1}{3}\pi r^2 h$ where $\frac{1}{3}\pi r^2 h = 120\text{ cm}^3$

Volume of cylinder $= \pi r^2 h$

$= 3 \times \frac{1}{3}\pi r^2 h$

$= 3 \times 120\text{ cm}^3 = \mathbf{360\text{ cm}^3}$

(ix) The solution set for the given inequation is : $-8 \le 2x < 8, x \in W$

(a) {–4, –3, –2, –1, 0, 1, 2, 3, 4}

(b) {–4, –3, –2, –1} (c) {0, 1, 2, 3}

(d) {–8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8}

Ans. (c) {0, 1, 2, 3}

Explanation :

$-8 \le 2x < 8, x \in W$

$\Rightarrow -4 \le x < 4, x \in W$ [Dividing each term by 2]

$\Rightarrow 0 \le x < 4$ as $x \in W$

$\therefore$ **The required solution is {0, 1, 2, 3}.**

(x) The probability of the sun rising from the east is P(S). The value of P(S) is :

(a) P(S) = 0 (b) P(S) < 0 (c) P(S) = 1 (d) P(S) > 1

Ans. (c) P(S) = 1

Explanation :

The sun always rises from the east so

S = Rising of the sun from east is a sure event and probability of a sure event is always 1 $\therefore$ **P(S) = 1**

(xi) If $\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 12 & 1 \end{bmatrix}$. The value of x *is* :

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (a) 2

Explanation :

$x + 3 \times 1 = 8 \quad \Rightarrow \quad \boldsymbol{x = 5}$

(xii) The centroid of a ΔABC is G(6, 7). If the coordinates of the vertices A, B and C are (a, 5), (7, 9) and (5, 7) respectively. The value of a is :

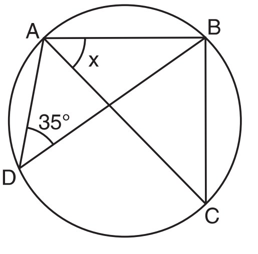
(a) 9 (b) 6 (c) 3 (d) 7

Ans. (b) 6

Explanation :

$$6 = \frac{a+7+5}{3} \quad \Rightarrow \boldsymbol{a = 6}$$

(xiii) In the given diagram AC is a diameter of the circle and $\angle$ADB = 35°. The degree measure of x is :



(a) 55°

(b) 35°

(c) 45°

(d) 70°

Ans. (a) 55°

Explanation :

$\angle$ACB = $\angle$ADB (Angles of same segment)

= 35°

$\angle$ABC = 90° (Angle of semi-circle)

In ΔABC, x + 35° + 90° = 180° $\Rightarrow$ $\boldsymbol{x = 55°}$

(xiv) If the nth term of an Arithmetic Progression (A.P.) is (n + 3), then the first three terms of the A.P. are :

(a) 1, 2, 3 (b) 2, 4, 6 (c) 4, 5, 6 (d) 7, 8, 9

Ans. (c) 4, 5, 6

Explanation :

$\because$ nth term of the A.P. = n + 3

its **1st term** = 1 + 3 = **4**

2nd term = 2 + 3 = **5**

and, **3rd term** = 3 + 3 = **6**

(xv) The median of a grouped frequency distribution is found graphically by drawing:

(a) a linear graph (b) a histogram

(c) a frequency polygon (d) a cumulative frequency curve

Ans. (d) a cumulative frequency curve

Explanation :

From a cumulative frequency curve only, we can locate a class, called median class, which contains the central value called median.

***Question* 2 :**

(i) Salman deposits ₹ 1200 every month in a recurring deposit account for $2\frac{1}{2}$ years. If the rate of interest is 6% per annum, find the amount he will receive on maturity. **[4]**

Ans. Given, monthly deposit = ₹ 1200 *i.e.* P = ₹ 1200

$$\text{Time} = 2\frac{1}{2} \text{ years} = \frac{5}{2} \times 12 \text{ months} = 30 \text{ months} \quad i.e \quad n = 30$$

and, rate of interest = 6% p.a. *i.e.* $r = 6\%$

∴ **Amount received on maturity** $= \text{P} \times n + \dfrac{\text{P}\times n\times(n+1)}{2\times12} \times \dfrac{r}{100}$

$$= ₹\ 1200 \times 30 + \frac{₹\,1200\times30\times31}{2\times12} \times \frac{6}{100}$$

= ₹ 38790

(ii) 3, 9, m, 81 and n are in continued proportion. Find the values of m and n. **[4]**

Ans. 3, 9, m, 81 and n are in continued proportion

$$\Rightarrow \frac{9}{3} = \frac{m}{9} = \frac{81}{m} = \frac{n}{81}$$

$$\Rightarrow \frac{9}{3} = \frac{m}{9} \text{ and } \frac{9}{3} = \frac{n}{81} \Rightarrow \boldsymbol{m = 27 \text{ and } n = 243}$$

(iii) Prove that : $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2 \sec A.$ **[4]**

Ans. **L.H.S.** $= \dfrac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} \qquad [\because \cos^2 A + \sin^2 A = 1]$$

$= \dfrac{2(1+\sin A)}{(1+\sin A)\cos A} = \dfrac{2}{\cos A} = 2 \sec A =$ **R.H.S.** **Hence proved.**

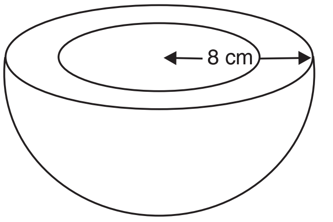
***Question* 3 :**

(i) The inner circumference of the rim of a circular metal tub is 44 cm. **[4]**

Find :

(a) The inner radius of the tub.

(b) The volume of the material of the tub if it's outer radius is 8 cm.



Use $\pi = \frac{22}{7}$

Give your answer correct to three significant figures.

Ans. (a) If inner radius of the tub is r cm

$2\pi r = 44 \Rightarrow \boldsymbol{r = 7}$ **cm**

(b) **Volume of the material of the tub**

$= \text{External volume} - \text{Internal volume}$

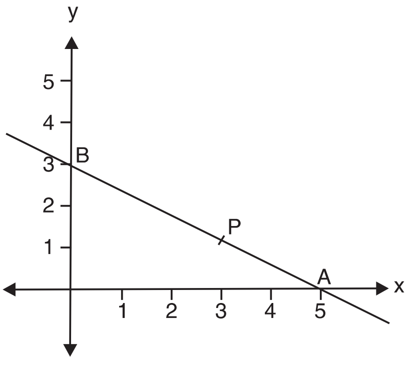
$= \frac{2}{3} \times \pi R^3 - \frac{2}{3} \times \pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (8^3 - 7^3)\ \text{cm}^3$

$= \frac{2}{3} \times \frac{22}{7} \times 169\ \text{cm}^3$

$= 354{\cdot}095\ \text{cm}^3 = \mathbf{354{\cdot}1\ cm^3}$

(ii) From the given figure : **[4]**



(a) Write down the coordinates of A and B.

(b) If P divides AB in the ratio 2 : 3, find the coordinates of point P.

(c) Find the equation of a line parallel to line AB and passing through origin.

Ans. (a) **Coordinates of A = (5, 0)** and

Coordinates of B = (0, 3)

(b) $P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$= \left(\frac{2\times0+3\times5}{2+3}, \frac{2\times3+3\times0}{2+3}\right)$

$= \left(\mathbf{3}, \mathbf{\frac{6}{5}}\right)$

$A(5, 0) = (x_1, y_1)$
$B(0, 3) = (x_2, y_2)$
and $m_1 : m_2 = 2 : 3$

(c) Slope of line AB = $\frac{3-0}{0-5} = -\frac{3}{5}$

Slope of the line parallel to AB = $-\frac{3}{5}$

Given, required line passes through origin (0, 0)

$\therefore$ Its equation is $y - 0 = -\frac{3}{5}(x - 0)$

$\Rightarrow$ $\mathbf{3x + 5y = 0}$

(iii) Use graph sheet for this question. Take 2 cm = 1 unit along the axes. Plot the ΔOAB, where O(0, 0), A(3, –2), B(2, –3). **[5]**

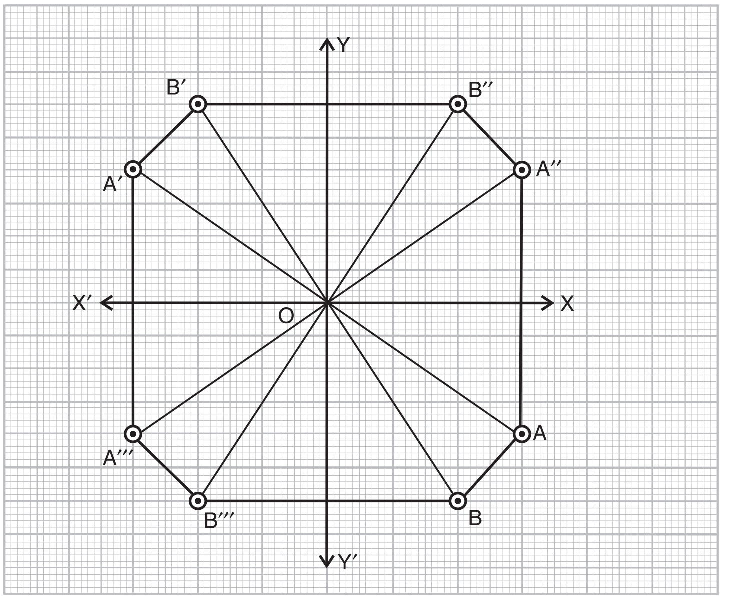
(a) Reflect the ΔOAB through the origin and name it as $\Delta OA'B'$.

(b) Reflect the $\Delta OA'B'$ on the y-axis and name it as $\Delta OA''B''$.

(c) Reflect the $\Delta OA'B'$ on the x-axis and name it as $\Delta OA'''B'''$.

(d) Join the points $AA''B''B'A'A'''B'''B$ and give the geometrical name of the closed figure so formed.

Ans.



The geometrical name of the figure obtained is **octagon**.

SECTION B

*(Attempt **any four** questions from this Section.)*

***Question* 4 :**

(i) The following bill shows the GST rates and the marked price of articles : **[3]**

BILL : COMPUTERS		
Articles	**Marked price**	**Rate of GST**
Graphic card	₹ 15500.00	18%
Laptop adapter	₹ 1900.00	28%

Find the total amount to be paid for the above bill.

Ans. The total amount to be paid for the given bill

= (₹ 15500 + 18% of ₹ 15500) + (₹ 1900 + 28% of ₹ 1900)

= ₹ 18290 + ₹ 2432 = **₹ 20722**

(ii) Solve the quadratic equation, $7x^2 + 2x - 2 = 0$. **[3]**

Give your answer correct to two places of decimal.

Ans. $7x^2 + 2x - 2 = 0$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \times 7 \times -2}}{2 \times 7}$$

$$= \frac{-2 \pm \sqrt{60}}{14} = \frac{-2 \pm 2\sqrt{15}}{14}$$

$$= \frac{-1 \pm \sqrt{15}}{7} = \frac{-1 \pm 3{\cdot}873}{7}$$

$$= \frac{-1 + 3{\cdot}873}{7} \text{ or } \frac{-1 - 3{\cdot}873}{7}$$

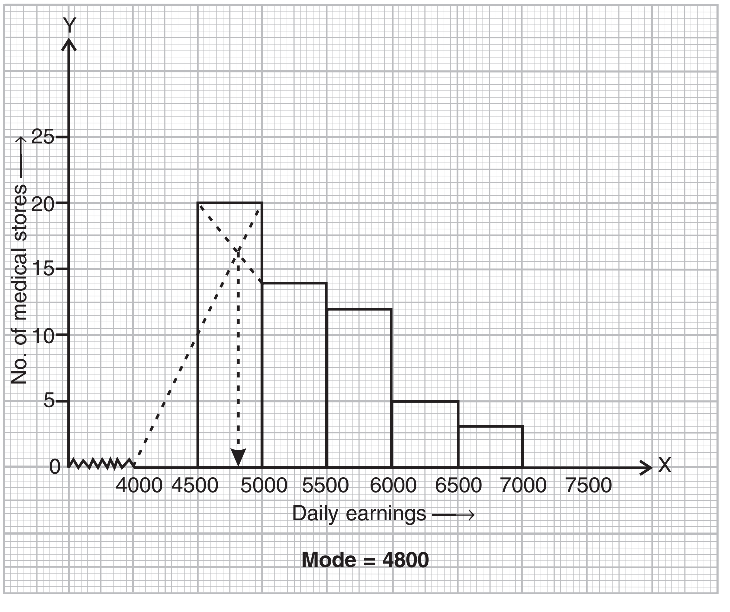
$= 0{\cdot}4104$ or $-0{\cdot}6961$

= **0·41** or **– 0·70**

(iii) Use graph sheet for this question. Draw a histogram for the daily earnings of 54 medical stores in the following table and hence estimate the mode for the following distribution. Take 2 cm = ₹ 500 units along the x-axis and 2 cm = 5 stores along the y-axis. **[4]**

Daily earnings (₹)	4500-5000	5000-5500	5500-6000	6000-6500	6500-7000
No. of medical stores	20	14	12	5	3

Ans. For all the given data, the graph will be as shown below :



***Question* 5 :**

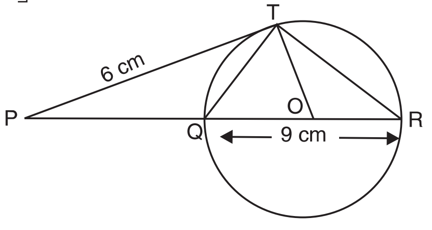
(i) $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$. Evaluate AB – 5C. **[3]**

Ans.
$$AB = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$$
$$5C = 5\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} -20 \\ 25 \end{bmatrix}$$
$$\therefore \quad \mathbf{AB - 5C} = \begin{bmatrix} 16 \\ -2 \end{bmatrix} - \begin{bmatrix} -20 \\ 25 \end{bmatrix} = \begin{bmatrix} \mathbf{36} \\ \mathbf{-27} \end{bmatrix}$$

(ii) In the given figure, O is the centre of circle. The tangent PT meets the diameter RQ produced at P. **[3]**

(a) Prove ΔPQT ~ ΔPTR.

(b) If PT = 6 cm, QR = 9 cm. Find the length of PQ.



Ans. (a) ∵ Angle between tangent and a chord through the point of contact is equal alternate opposite angle

∴ $\angle PTQ = \angle TRQ$

$\angle P = \angle P$ (Common)

⇒ **ΔPQT ~ ΔPTR** (By A.A.)

(b) $PT^2 = PQ \times PR$

⇒ $6^2 = x \times (x + 9)$ [Let PQ = x]

⇒ $x^2 + 9x - 36 = 0$

⇒ $x^2 + 12x - 3x - 36 = 0$ *i.e.* $x = 3$ or -12

⇒ $x = 3$ *i.e.* **PQ = 3 cm**

(iii) Factorise the given polynomial completely, using Remainder Theorem : **[4]**

$6x^3 + 25x^2 + 31x + 10$

Ans. For $x = -2$

Remainder $= 6 \times (-2)^3 + 25 \times (-2)^2 + 31 \times (-2) + 10$

$= -48 + 100 - 62 + 10 = 0$

∴ $(x + 2)$ is a factor of polynomial $6x^3 + 25x^2 + 31x + 10$

$$\begin{array}{r|l} & 6x^2 + 13x + 5 \\ x+2 & 6x^3 + 25x^2 + 31x + 10 \\ & 6x^3 + 12x^2 \\ & - \quad - \\ \hline & 13x^2 + 31x \\ & 13x^2 + 26x \\ & - \quad - \\ \hline & 5x + 10 \\ & 5x + 10 \\ & - \quad - \\ \hline & \times \end{array}$$

∴ $\mathbf{6x^3 + 25x^2 + 31x + 10}$

$= (x + 2)(6x^2 + 13x + 5)$

$= (x + 2)(6x^2 + 10x + 3x + 5)$

$= (x + 2)[2x(3x + 5) + 1(3x + 5)]$

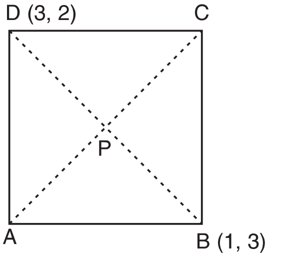
$= \mathbf{(x + 2)(3x + 5)(2x + 1)}$

***Question* 6 :**

(i) ABCD is a square where B(1, 3), D(3, 2) are the end points of the diagonal BD. Find : **[3]**

(a) the coordinates of point of intersection of the diagonals AC and BD.

(b) the equation of the diagonal AC.



Ans. (a) The co-ordinates of point of intersection of the diagonals AC and BD.

Point P = Mid-point of BD

$$= \left(\frac{1+3}{2}, \frac{3+2}{2}\right) = \left(\mathbf{2, \frac{5}{2}}\right)$$

(b) Slope of diagonal BD = $\frac{3-2}{1-3} = -\frac{1}{2}$

Slope of diagonal AC = 2

[Diagonals of a square bisect each other at right angle]

For equation of diagonal AC; slope $m = 2$

and point = $P\left(2, \frac{5}{2}\right) = (x_1, y_1)$

∴ **Equation of AC** is : $y - \frac{5}{2} = 2(x - 2)$

⇒ $\mathbf{4x - 2y = 3}$

(ii) Prove that : $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \sec\theta \cdot \operatorname{cosec}\theta$. **[3]**

Ans. L.H.S. $= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$

$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}}$$

$$= \sqrt{\frac{1}{\cos^2\theta\sin^2\theta}} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= \frac{1}{\cos\theta\sin\theta}$ = **sec θ·cosec θ = R.H.S.** **Hence proved.**

(iii) The first, the last term and the common difference of an Arithmetic Progression are 98, 1001 and 7 respectively. Find the following for the given Arithmetic Progression : **[4]**

(a) Number of terms 'n'.

(b) Sum of the 'n' terms.

Ans. For the given Arithmetic Progression : $a = 98$, $l = 1001$ and $d = 7$

(a) $l = 1001$

$\Rightarrow \quad a + (n - 1)\, d = 1001 \quad i.e. \quad 98 + (n - 1) \times 7 = 1001$

$\Rightarrow \quad (n - 1) \times 7 = 903 \quad i.e. \quad n - 1 = 129$

$\Rightarrow \quad \boldsymbol{n = 130}$

(b) **Sum of the '*n*' terms** $= \frac{n}{2}(a + l)$

$= \frac{130}{2}[98 + 1001]$

$= 65 \times 1099 = \mathbf{71435}$

***Question* 7 :**

(i) A box contains some green, yellow and white tennis balls. The probability of selecting a green ball is $\frac{1}{4}$ and yellow ball is $\frac{1}{3}$. If the box contains 10 white balls, then find : **[3]**

(a) total number of balls in the box.

(b) probability of selecting a white ball.

Ans. (a) $\because$ Sum of the probabilities of all events = 1

$\Rightarrow$ P(green ball) + P(yellow ball) + P(white ball) = 1

$\Rightarrow \frac{1}{4} + \frac{1}{3}$ + P(white ball) = 1

$\Rightarrow$ P (white ball) $= \frac{5}{12}$

If total number of balls in the box are x, P(white ball) $= \frac{10}{x}$

$\Rightarrow \frac{10}{x} = \frac{5}{12} \quad i.e. \quad x = 24$

$\therefore$ **Number of balls in the box = 24**

(b) **Probability of selecting a white ball** $= \mathbf{\frac{5}{12}}$

(ii) A cone and a sphere having the same radius are melted and recast into a cylinder. The radius and height of the cone are 3 cm and 12 cm respectively. If the radius of the cylinder so formed is 2 cm, find the height of the cylinder. **[3]**

Ans. According to the given statement

Volume of cone + Volume of sphere = Volume of cylinder

$\Rightarrow \frac{1}{3} \times \pi \times 3^2 \times 12 + \frac{4}{3} \times \pi \times 3^3 = \pi \times 2^2 \times h$

$\Rightarrow \frac{1}{3} \times 3^2 \times 12 + \frac{4}{3} \times 3^3 = 2^2 \times h$

$\Rightarrow 36 + 36 = 4h \quad i.e. \quad \boldsymbol{h = 18}$ **cm**

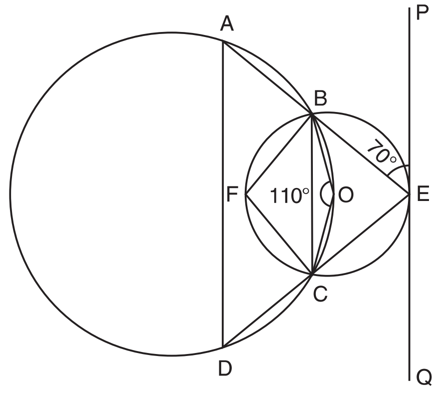
(iii) In the given diagram, ABCD is a cyclic quadrilateral and PQ is a tangent to the smaller circle at E. Given $\angle AEP = 70°$, $\angle BOC = 110°$. Find : **[4]**

(a) $\angle ECB$

(b) $\angle BEC$

(c) $\angle BFC$

(d) $\angle DAB$



Ans. (a) For the circle with centre O,

$\boldsymbol{\angle ECB} = \angle BEP$ (*i.e.* $\angle AEP$)

$= \mathbf{70°}$

(b) $\angle AOC = 2\angle BEC$

$\Rightarrow 110° = 2\angle BEC$ *i.e.* $\boldsymbol{\angle BEC = 55°}$

(c) $\angle BFC + \angle BEC = 180°$

$\Rightarrow \angle BFC + 55° = 180°$ *i.e.* $\boldsymbol{\angle BFC = 125°}$

(d) In cyclic quadrilateral ABCD,

ext $\angle BCE = \angle DAB$

$\Rightarrow \boldsymbol{\angle DAB = 70°}$

***Question* 8 :**

(i) Solve the following inequation : **[3]**

$$-\frac{x}{3} - 4 \le \frac{x}{2} - \frac{7}{3} < -\frac{7}{6},\ x \in R$$

Represent the solution set on a number line.

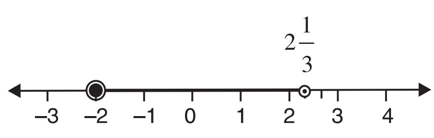
Ans. Since L.C.M. of denominators 3, 2, 3 and 6 is 6; multiply each term of the given inequation by 6 to get :

$-2x - 24 \le 3x - 14 < -7,\ x \in R$

$-2x - 24 \le 3x - 14$	$3x - 14 < -7$
$\Rightarrow 5x \ge -10$	$\Rightarrow 3x < 7$
$\Rightarrow x \ge -2$	$\Rightarrow x < 2\frac{1}{3}$

$\therefore$ Solution $= \{x : -2 \le x < 2\frac{1}{3}\}$

Solution set on number line is :



(ii) The following table gives the petrol prices per litre for a period of 50 days. **[3]**

Price (₹)	85-90	90-95	95-100	100-105	105-110
No. of days	12	10	8	15	5

Find the mean price of petrol per litre to the nearest rupee using step – deviation method.

Ans.

Price (₹) C.I.	No. of days f	Class marks x	If assumed mean (A) = 97·5 $u = \frac{x - A}{i}$	$f \times u$
85-90	12	87·5	–2	–24
90-95	10	92·5	–1	–10
95-100	8	97·5	0	0
100-105	15	102·5	1	15
105-110	5	107·5	2	10
	$\Sigma f = 50$			$\Sigma f \times u = -9$

Mean price of petrol per litre

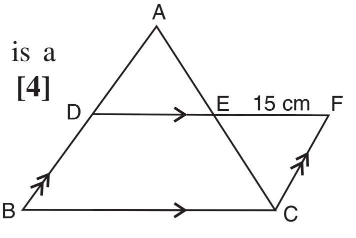
$$= A + \frac{\sum fu}{\sum f} \times i = ₹ \left[97{\cdot}5 + \frac{-9}{50} \times 5\right]$$

$$= ₹ (97{\cdot}5 - 0{\cdot}9) = ₹ 96{\cdot}60 = \mathbf{₹ 97}$$

(iii) In the given diagram, ABC is a triangle and BCFD is a parallelogram. AD : DB = 4 : 5 and EF = 15 cm. **[4]**

Find :

(a) AE : EC (b) DE (c) BC



Ans. (a) In ΔABC, DE is parallel to BC

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB} = \frac{4}{5} \quad \textit{i.e.} \quad \mathbf{AE : EC = 4 : 5}$$

(b) ΔADE ~ ΔCFE by A.A.A.

$$\Rightarrow \frac{DE}{EF} = \frac{AE}{EC} \quad \textit{i.e.} \quad \frac{DE}{15} = \frac{4}{5} \quad \Rightarrow \mathbf{DE = 12\ cm}$$

(c) **BC** = DF

= DE + EF = 12 cm + 15 cm = **27 cm**

***Question* 9 :**

(i) Amit takes 12 days less than the days taken by Bijoy to complete a certain work. If both, working together, take 8 days to complete the work, find the number of days taken by Bijoy to complete the work, working alone. **[4]**

Ans. Let Bijoy takes x days to complete the work

⇒ Amit takes $(x - 12)$ days to complete the same work

According to the given statement :

$$\frac{1}{x-12} + \frac{1}{x} = \frac{1}{8}$$

$$\Rightarrow \frac{x + x - 12}{x(x-12)} = \frac{1}{8} \quad \Rightarrow x^2 - 12x = 16x - 96$$

$$\Rightarrow x^2 - 28x + 96 = 0$$

$$\Rightarrow x^2 - 24x - 4x + 96 = 0 \quad \textit{i.e.} \quad x = 24 \text{ or } x = 4$$

$x = 4$ does not satisfy the given condition

$\therefore \quad x = 24$

⇒ **No. of days taken by Bijoy to complete the work = 24**

(ii) Use a graph sheet for this question. The daily wages of 120 workers working at a site are given below : **[6]**

Wages (₹)	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of workers	8	15	20	30	25	15	7

Use 2 cm = ₹ 50 and 2 cm = 20 workers along x-axis and y-axis respectively to draw an ogive and hence estimate :

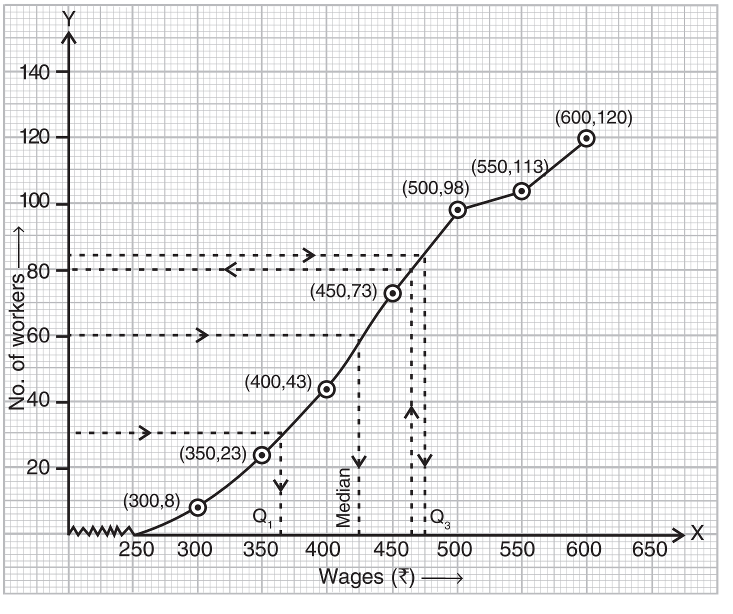
(a) the median wages (b) the inter-quartile range of wages

(c) percentage of workers whose daily wage is above ₹475

Ans.

Wages (₹) (C.I.)	**No. of workers (f)**	**Cumulative frequency (C.F.)**
250-300	8	8
300-350	15	23
350-400	20	43
400-450	30	73
450-500	25	98
500-550	15	113
550-600	7	120

The ogive obtained for the given data is as given below :



(a) **The median wages** = **₹ 425**

(b) **The inter-quartile range of wages** = $Q_3 - Q_1$

= ₹ 485 – ₹ 365 = **₹ 120**

(c) ∵ No. of workers with daily wages ₹ 475 = 82

∴ No. of workers with daily wages above ₹ 475 = 120 – 80 = 40

And, **percentage of workers whose daily wages is above ₹ 475**

$$= \frac{40}{120} \times 100\% = \mathbf{33{\cdot}3\%}$$

Question **10 :**

(i) Solve for x, using the properties of proportion : $\dfrac{\sqrt{2+x}+\sqrt{3-x}}{\sqrt{2+x}-\sqrt{3-x}} = 3.$ **[3]**

Ans. Applying componendo and dividendo, we get :

$$\frac{\sqrt{2+x}+\sqrt{3-x}+\sqrt{2+x}-\sqrt{3-x}}{\sqrt{2+x}+\sqrt{3-x}-\sqrt{2+x}+\sqrt{3-x}} = \frac{3+1}{3-1}$$

$$\Rightarrow \quad \frac{2\sqrt{2+x}}{2\sqrt{3-x}} = \frac{4}{2} = 2 \quad i.e. \quad \frac{\sqrt{2+x}}{\sqrt{3-x}} = 2 \quad \Rightarrow \frac{2+x}{3-x} = 4$$

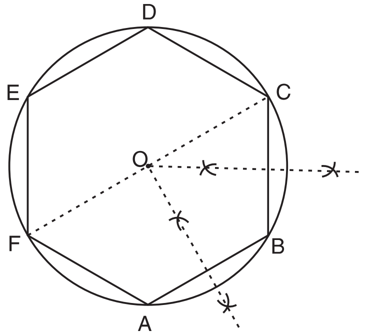
Again applying componendo and dividendo, we get :

$$\frac{2+x+3-x}{2+x-3+x} = \frac{4+1}{4-1} \quad i.e. \quad \frac{5}{2x-1} = \frac{5}{3} \quad \Rightarrow 2x - 1 = 3 \text{ and } \boldsymbol{x = 2}$$

(ii) Using ruler and compasses, construct a regular hexagon of side 4·5 cm. Hence construct a circle circumscribing the hexagon. Measure and write down the length of the circum-radius. **[3]**

Ans. $\because$ Each interior angle of a regular hexagon = $\dfrac{(2\times 6-4)}{6} \times 90° = 120°$

Draw the required regular hexagon ABCDEF with AB = BC = CD = DE = EF = FA = 4·5 cm and ∠A = ∠B = ∠C = ∠D = ∠E = ∠F = 120°



Draw perpendicular bisectors of any two consecutive sides say AB and BC. These two perpendicular bisectors meet each other at point O which is centre of required circle circumscribing the hexagon obtained. With point O as centre and OA (or OB, or OC, or OD, or OE, or OF) as radius draw the circle which passes through vertices A, B, C, D, E and F. It is the required circle circumscribing the hexagon ABCDEF.

(iii) An observer standing on the top of a lighthouse 150 m above the sea level watches a ship sailing away. As he observes, the angle of depression of the ship changes from 50° to 30°. Determine the distance travelled by the ship during the period of observation. Give your answer correct to the nearest meter. (Use Mathematical Table for this question). **[4]**

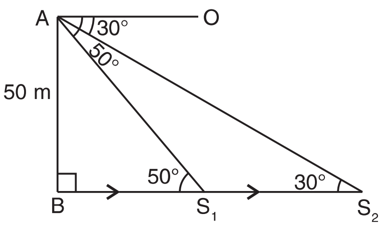
Ans. According to the given statement the diagram will be as shown below.

Clearly, AB = height of lighthouse = 150 m

S_1 and S_2 are two positions of the ship,

$\angle AS_1B = \angle OAS_1 = 50°$ and

$\angle AS_2B = \angle OAS_2 = 30°$



In ΔABS_1, $\tan 50° = \dfrac{150}{BS_1} \Rightarrow 1{\cdot}1918 = \dfrac{150}{BS_1}$

$\Rightarrow BS_1 = \dfrac{150}{1{\cdot}1918}$ m = 125·86 m.

In ΔABS_2, $\tan 30° = \dfrac{150}{BS_2} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{150}{BS_2}$

$\Rightarrow BS_2 = 150 \times 1{\cdot}732$ m = 259·8 m

$\therefore$ **Required distance** = $BS_2 - BS_1$= 259·8 m – 125·86 m = 133·94 m = **134 m**